JOURNAL OF MATERIALS SCIENCE 18 (1983) 1323–1330

# Numerical determination of the parameters of Krupkowski's function for a torsion test taking into consideration the strain hardening ranges

A. GRABIANOWSKI
*Institute for Metal Research, Polish Academy of Sciences, Kraków, Poland*

J. KLOCH
*Institute of Mathematics, Polish Academy of Sciences, Kraków, Poland*

A method has been proposed for determining the parameters of a function describing the strain hardening curve for a torsion test taking into consideration the strain hardening ranges. Certain measures for the approximation quantity have been defined by means of which it is possible to formulate the criterion of the occurrence of these ranges. Experimental verification of the method was carried out using nickel of 99.95% purity. Test pieces of a circular cross-section were subjected to a torsion test at ambient temperature without taking into consideration the strain hardening rate.

## Nomenclature

| | |
|---|---|
| $\tau(\gamma_R)$ | shear stress of the metal deformed up to $\gamma_R$. |
| $\gamma_R$ | shear strain at a distance $R$ from the centre of the test-piece. |
| $d$ | diameter of the test-piece undergoing torsion, $d = 2R$. |
| $l$ | length of the test-piece. |
| $\alpha$ | angle of torsion. |
| $M$ | torsion moment. |
| $\gamma^1; k; m; c$ | material parameters. |
| $\sigma$ | effective stress. |
| $z$ | measure of deformation in form of actual cold work. |
| $z^1$ | initial deformation equivalent of latent cold work. |
| $\mathrm{d}L^0$ | the elementary work of deformation per unit of volume. |
| $\gamma_{I/II}$ | boundary between the strain hardening ranges I and II. |
| $\gamma_{II/III}$ | boundary between the strain hardening ranges II and III. |
| A; H; S; SS; B | approximation quantity measures described in the Section 3. |

## 1. Introduction

The results of a torsion test are generally interpreted on the basis of Duguet's formula [1]

$$\tau(\gamma_R) = \frac{1}{2\pi R^3}\left(3M + \gamma_R \frac{\mathrm{d}M}{\mathrm{d}\gamma_R}\right) \quad (1)$$

where

$$\gamma_R = \frac{R\alpha[\mathrm{rad}]}{l}$$

In result, when describing the deformation geometry by a scheme as in Fig. 1, we obtain the characteristics of the examined metal in the form of a strain hardening curve.

Krupkowski [2] proposed a different approach to this problem, and he suggested that the strain hardening process in the torsion test should be interpreted by means of the dependence

$$\tau = ck[1 - \exp(-c\gamma_i)]^m \quad (2)$$

where $\gamma_i = \gamma^1 + \gamma$. This function contains the material parameters: $\gamma^1; k; m; c$, to which physical meaning is ascribed.

0022–2461/83 $03.00 + .12 

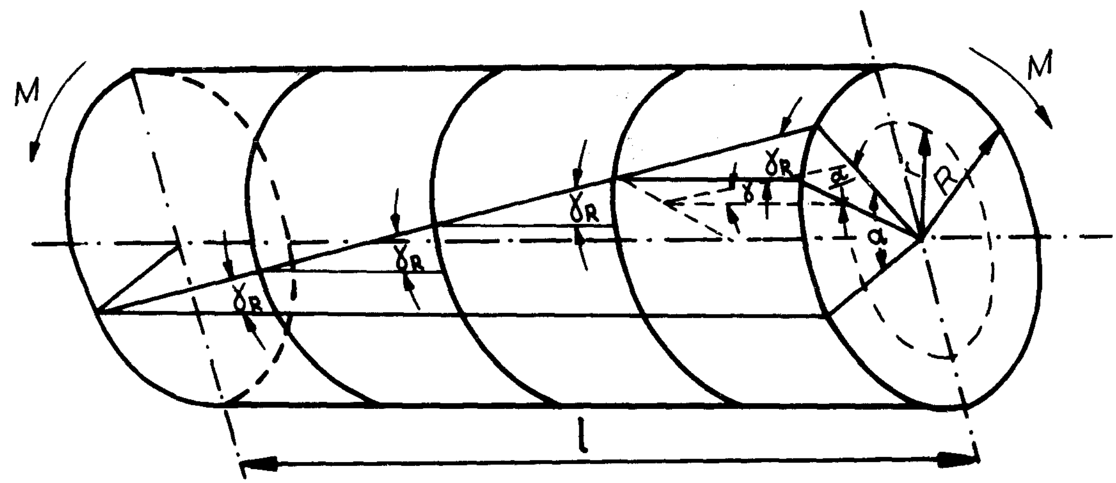


*Figure 1* The assumed strain hardening model of a test-piece subjected to torsion.

When formulating Equation 2 Krupkowski also assumed the deformation geometry shown in Fig. 1, and next he adopted the concept that the deformed metal is characterized by two properties which are determined by the extrapolation of the strain hardening curve: $\tau^1 = ck[1 - \exp(-c\gamma^1)]^m$ for $\gamma_i = \gamma^1$ (i.e. for a latent value) and $\tau_{max} = ck$ for $\gamma \to \infty$. The parameter $m$ in this case indicates the capacity for the strain hardening of the metal.

It should be noted that Equation 2 has been derived from Krupkowski's formula [3]

$$\sigma = k[z^1 + (1 - z^1)z]^m \tag{3}$$

based on the deformation measures $z$ established on the basis of the principle of an identical unit work of plastic deformation $dL^0$, thus:

$$\text{for torsion} \qquad z = 1 - e^{-c\gamma_i}$$

$$\text{for tension} \qquad z = 1 - \frac{A}{A_0}$$

$$\text{for compression} \quad z = 1 - \frac{A_0}{A} \tag{4}$$

where $A_0$, $A$ denote the initial and final cross-section of the test-piece.

The above dependencies are valid for the case when the deformation of the metal is homogeneous.

When analysing Equations 2 and 3 it can be seen that if $c = \tau/\sigma =$ constant, then with increasing $z$ the parameters $k$ and $m$ retain the same meaning. Thus we may adopt the concept [4] that the unit hardening work

$$dL^0 = -kz_i^m d\ln(1 - z_i) \tag{5}$$

where $z_i = z^1 + (1 - z^1)z$ does not depend on the deformation method and is characteristic for the given metal. As a result when applying Equations 2 and 3 it is possible, basing on the given torsion, tension and compression tests to determine one curve for the strain hardening of the metal.

We must remember that Equation 2 is a relation which is valid at a definite shear strain rate $\dot{\gamma}_i$ and temperature $T$ of the deformation of the metal, to which there corresponds a certain intersection of the hypersurface of the plastic state $\tau = \tau(\gamma_i, \dot{\gamma}_i, T)$ for $T =$ constant, and $\dot{\gamma}_i =$ constant (Fig. 2).

This function has already been developed by other authors and new terms have been added which take into consideration also the effects of

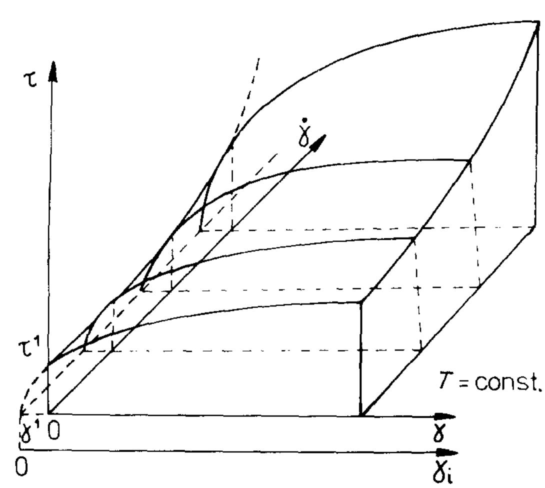


*Figure 2* Surface scheme of plastic state in case of generalized Krupkowski's function.

$T$ and $\dot{\gamma}$. The present publication, however, is restricted to the discussion only of the effect of deformation.

Investigations of the strain hardening of various metals have shown that within a wide range of deformations the nonhomogeneous deformation is the prevailing one, and consequently, the strain hardening curves should be separated into ranges in which the material parameters retain constant values [5, 6].

Thus, independently of the above quoted parameters, when discussing the torsion tests, it is necessary to determine two additional parameters, namely the boundaries between the strain hardening ranges: $\gamma_{I/II}$; $\gamma_{II/III}$. This is not an easy task, especially when dealing with the results of a torsion test of a solid test-piece where metal cores belonging to different strain hardening ranges, which undergo simultaneous deformation, occur [7, 8]. Consequently, when interpreting the torsion by means of Equation 2 we must solve the following problems:

(a) establish the criteria for the occurrence of the ranges,
(b) provide the method of determining the boundaries between the ranges,
(c) provide the method of determining the values of the parameters: $k$; $c$; $\gamma^1$; $m$ within the particular ranges.

Moreover, there arises the question whether the values of all the determined parameters within the particular ranges should be different.

Basing on [9] it has been assumed in this paper that the value of the parameter $\gamma^1$ is characteristic for the initial state of the metal, and thus it remains constant over all the ranges.

The principal aim of this publication is to present the method proposed for the solution of the problems mentioned as items (a) to (c) above. It consists of a very accurate approximation of experimental data based on the generalized least squares method, the rectangles of the measurement errors and the accuracy of the related original measurements. These considerations were the basis for the elaboration of an appropriate digital computer program which was used to carry out the first calculations.

## 2. Formulating the problem

According to the results presented in [1] the relation between the torsion moment of a solid test-piece with a circular cross-section $M = M(\alpha)$ and the shear stress $\tau = \tau(r, \alpha)$ can be described by the formula

$$M(\alpha) = 2\pi \int_0^{d/2} r^2 \tau(r, \alpha)\, \mathrm{d}r \qquad (6)$$

where $\alpha$ is the angle of torsion measured in degrees, and $r$ is the radial coordinate (Fig. 1).

$$\tau(r, \alpha) = ck\left\{1 - \exp\left[-c\left(\gamma^1 + \frac{\pi r \alpha}{l \times 180^\circ}\right)\right]\right\}^m \qquad (7)$$

If in Equation 6 we substitute $r = xd/2$, where $x \in [0, 1]$, and next assume the notation

$$\gamma = \frac{\pi d \alpha}{l \times 360^\circ}, \qquad \bar{M}(\gamma) = \frac{4M(\alpha)}{\pi d^3} \qquad (8)$$

then we shall obtain

$$\bar{M}(\gamma) = ck \int_0^1 x^2\{1 - \exp[-c(\gamma^1 + x\gamma)]\}^m \mathrm{d}x \qquad (9)$$

The above formula defines the form of a functional relationship between the quantities $\gamma$ and $\bar{M}(\gamma)$ and in this way between the direct results of measurement of $(\alpha_j, M_j)\, j = 1, 2, \ldots, n$ and it will constitute the starting point for a detailed formulation of the programme.

While a test-piece is undergoing torsion the quantities: $d$, $l$, $(\alpha_j, M_j)$ are determined by direct and independent measurements. The measurement errors of $d, l, \alpha_j$ are constant and equal $\Delta d$, $\Delta l$ and $\Delta\alpha$, respectively. The measurement error of $M_j$ equals $s$%, hence $\Delta M_j = 0.01sM_j$. Knowing $\Delta d, \Delta l$, $\Delta\alpha$, $\Delta M_j$ we can make use of Equation 8 and of the law of the propagation of error and calculate

$$\Delta\gamma_j = \left[\left(\frac{\partial\gamma}{\partial d}\right)^2\bigg|_{x_j}(\Delta d)^2 + \left(\frac{\partial\gamma}{\partial l}\right)^2\bigg|_{x_j}(\Delta l)^2 + \left(\frac{\partial\gamma}{\partial\alpha}\right)^2\bigg|_{x_j}(\Delta\alpha)^2\right]^{1/2},$$

$$\Delta\bar{M}_j = \left[\left(\frac{\partial\bar{M}}{\partial d}\right)^2\bigg|_{x_j}(\Delta d)^2 + \left(\frac{\partial\bar{M}}{\partial M}\right)^2\bigg|_{x_j}(\Delta M_j)^2\right]^{1/2},$$

For simplicity of description let us introduce the following notation:

$$\omega_j = (\Delta\bar{M}_j)^{-2}, \qquad X_j = (d, l, \alpha_j, M_j). \qquad (10)$$

In case of a single strain hardening range our problem can be formulated as follows: having a series of points $(\gamma_j, \bar{M}_j)\, j = 1, 2, \ldots, n$ and the function

$$U(a,b,c,k,m,\gamma^1,\gamma) = U(a,b,X,\gamma^1,\gamma)$$
$$= ck\int_a^b x^2\{1-\exp[-c(\gamma^1+x\gamma)]\}^m \mathrm{d}x \tag{11}$$

find such values of the parameters $(c_{\mathrm{I}},k_{\mathrm{I}},m_{\mathrm{I}},\gamma_{\mathrm{I}}^1) = (X_{\mathrm{I}},\gamma_{\mathrm{I}}^1)$ that the function

$$F_{1,n}(X,\gamma^1) = \sum_1^n \omega_j\,[M_j - U(0,1,X,\gamma^1,\gamma_j)]^2$$

reaches its global minimum at the point $(X_{\mathrm{I}},\gamma_{\mathrm{I}}^1)$. Substituting in Equation 11: $a=0$, $b=1$, $(c,k,m,\gamma^1) = (c_{\mathrm{I}},k_{\mathrm{I}},m_{\mathrm{I}},\gamma_{\mathrm{I}}^1)$ we shall obtain a function describing the strain hardening process in the metal, for which a single strain hardening range has been derived.

Let $(\gamma_p,\bar{M}_p)$ be one of the points $(\gamma_j,\bar{M}_j)$. By establishing the point $(\gamma_p,\bar{M}_p)$ we have divided the series of the points $(\gamma_j,\bar{M}_j)$ into two ranges: the first composed of the points $(\gamma_j,\bar{M}_j)$ $j=1,\ldots,p$, the second – of the points $(\gamma_j,\bar{M}_j)$ $j=p+1,\ldots,n$.

If for a given metal we introduce two strain hardening ranges, then the value of all the parameters and the boundary between the ranges will be determined in the following sequence: for the initial $p$ points we find the values of the parameters $(c_p,k_p,m_p,\gamma_p^1) = (X_p,\gamma_p^1)$ for which the function $F_{1,p}(X,\gamma^1)$ attains its minimum.

Within the second range the core of the test-piece, the diameter of which changes with the change of $\gamma$ and is equal to $d\gamma_p/\gamma_j$ [3] becomes hardened according to the laws valid in the first range. For this reason before determining the strain hardening parameters within the second range we must calculate the values $\bar{M}_{1,j} = \bar{M}_j - U(0,\gamma_p/\gamma_j,X_p,\gamma_p^1,\gamma_j)$ where $j=p+1,\ldots,n$.

Subsequently, for the series of the points $(\gamma_j,\bar{M}_{1,j})$ $j=p+1,\ldots,n$ we find such values of $(c_n,k_n,m_n) = X_n$ for which the function

$$F_{p+1,n}(X) = \sum_{p+1}^n \omega_j[\bar{M}_{1,j} - U(\gamma_p/\gamma_j,1,X,\gamma_p^1,\gamma_j)]^2$$

attains its minimum.

In the end we determine $p=\bar{p}$ for which the function

$$F(p) = F_{1,p}(X_p,\gamma_p^1) + F_{p+1,n}(X_n)$$

attains its minimum.

Denoting $X_{\mathrm{I}} = (c_{\bar{p}},k_{\bar{p}},m_{\bar{p}})$, $\gamma_{\mathrm{I}}^1 = \gamma_{\bar{p}}^1$, $X_{\mathrm{II}} = (c_n,k_n,m_n)$ $\gamma_{\mathrm{I/II}} = \gamma_{\bar{p}}$ we can write down the function describing the strain hardening process of the metal for which two strain hardening ranges have been derived, in the following form:

$$\bar{M}(\gamma) = \begin{cases} U(0,1,X_{\mathrm{I}},\gamma_{\mathrm{I}}^1,\gamma), & \gamma\in[0,\gamma_{\mathrm{I/II}}] \\ U(0,\gamma_{\mathrm{I/II}}/\gamma,X_{\mathrm{I}},\gamma_{\mathrm{I}}^1,\gamma) \\ \quad + U(\gamma_{\mathrm{I/II}}/\gamma,1,X_{\mathrm{II}},\gamma_{\mathrm{I}}^1,\gamma), \\ \quad \gamma\in[\gamma_{\mathrm{I/II}},\gamma_n] \end{cases} \tag{12}$$

By establishing two points $(\gamma_p,\bar{M}_p)$, $(\gamma_q,\bar{M}_q)$ from the series $(\gamma_j,\bar{M}_j)$, $1<p<q<n$, we divide it into three ranges; the first, composed of the points $(\gamma_j,\bar{M}_j)$ $j=1,\ldots,p$; the second, composed of the points $(\gamma_j,\bar{M}_j)$ $j=p+1,\ldots,q$; the third, of the points $(\gamma_j,\bar{M}_j)$ $j=q+1,\ldots,n$.

If for a given metal there are introduced three strain hardening ranges, then, when discussing the first two ranges, we proceed analogously as in the case of two ranges, described above. Here we obtain the quantities: $(X_p,\gamma_p^1)$, $X_q$, $F_{1,p}(X_p,\gamma_p^1)$ and $F_{p+1,q}(X_q)$.

Within the third range, besides the core of the test-piece which becomes hardened according to the laws valid over the first range, we must take into consideration the layer whose thickness is a function of $\gamma$ [3] becoming hardened according to the laws valid over the second range. Hence, before determining the strain hardening parameters in the third range we shall calculate the values:

$$\bar{M}_{2,j} = \bar{M}_{1,j} - U(\gamma_p/\gamma_j,\gamma_q/\gamma_j,X_q,\gamma_p^1,\gamma_j),$$
$$j = q+1,\ldots,n.$$

Next, for the series of points $(\gamma_j,\bar{M}_{2,j})$ we find such values of the parameters $(c_n,k_n,m_n) = X_n$ for which the function

$$F_{q+1,n}(X) = \sum_{q+1}^n \omega_j\,[\bar{M}_{2,j} - U(\gamma_q/\gamma_j,1,X,\gamma_p^1,\gamma_j)]^2$$

attains its minimum.

To determine the boundaries between the ranges we find such $p=\bar{p}$ and $q=\bar{q}$ for which the function

$$F(p,q) = F_{1,p}(X_p,\gamma_p^1) + F_{p+1,q}(X_q) + F_{q+1,n}(X_n)$$

attains its minimum.

Denoting $X_{\mathrm{I}} = (c_{\bar{p}},\ k_{\bar{p}},\ m_{\bar{p}})$, $\gamma_{\mathrm{I}}^1 = \gamma_{\bar{p}}^1$, $X_{\mathrm{II}} = (c_{\bar{q}},\ k_{\bar{q}},\ m_{\bar{q}})$, $\gamma_{\mathrm{I/II}} = \gamma_{\bar{p}}$, $X_{\mathrm{III}} = (c_n,\ k_n,\ m_n)$, $\gamma_{\mathrm{II/III}} = \gamma_{\bar{q}}$ we can express the function describing the strain hardening process of the metal for which three strain hardening ranges have been derived in the following form:

$$\bar{M}(\gamma) = \begin{cases} U(0,1,X_{\mathrm{I}},\gamma_{\mathrm{I}}^1,\gamma), & \gamma \epsilon\, [0,\gamma_{\mathrm{I/II}}] \\ U(0,\gamma_{\mathrm{I/II}}/\gamma,X_{\mathrm{I}},\gamma_{\mathrm{I}}^1,\gamma) & \\ +\,U(\gamma_{\mathrm{I/II}}/\gamma,1,X_{\mathrm{II}},\gamma_{\mathrm{I}}^1,\gamma) & \\ \quad \gamma \epsilon\, [\gamma_{\mathrm{I/II}},\gamma_{\mathrm{II/III}}] & \\ U(0,\gamma_{\mathrm{I/II}}/\gamma,X_{\mathrm{I}},\gamma_{\mathrm{I}}^1,\gamma) & \\ +\,U(\gamma_{\mathrm{I/II}}/\gamma,\gamma_{\mathrm{II/III}}/\gamma,X_{\mathrm{II}},\gamma_{\mathrm{I}}^1,\gamma) & \\ +\,U(\gamma_{\mathrm{II/III}}/\gamma,1,X_{\mathrm{III}},\gamma_{\mathrm{I}}^1,\gamma), & \\ \quad \gamma \epsilon\, [\gamma_{\mathrm{II/III}},\gamma_n]\,. & \end{cases} \tag{13}$$

## 3. Approximation quantity measures

Let us denote: $\gamma_j^+ = \gamma_j + \Delta\gamma_j$, $\gamma_j^- = \gamma_j - \Delta\gamma_j$, $\bar{M}_j^+ = \bar{M}_j + \Delta\bar{M}_j$, $\bar{M}_j^- = \bar{M}_j - \Delta\bar{M}_j$, $M_j^{\mathrm{T}} = \bar{M}(\gamma_j)$, where $j = 1, \ldots, n$. Depending on the fact whether one, two or three ranges are introduced, the function $\bar{M}(\gamma_j)$ is defined by Equation 11, 12 or 13, respectively.

A rectangle $R_j$ with the vertices $(\gamma_j^-,\ \bar{M}_j^-)$, $(\gamma_j^+,\ \bar{M}_j^-)$, $(\gamma_j^+,\ \bar{M}_j^+)$, $(\gamma_j^-,\ \bar{M}_j^+)$ (Fig. 3) is called the error rectangle.

The approximation quantity measures presented below of the points $(\gamma_j, \bar{M}_j)$ by means of function $\bar{M}(\gamma)$ are helpful in making the decision whether for the given metal we should introduce one, two or three strain hardening ranges.

(a) Function

$$E(\gamma_j) = \min(\bar{M}_j^+ - \bar{M}(\gamma_j^-), \bar{M}(\gamma_j^+) - \bar{M}_j^-)$$

The values of this function inform us about the position of the curve of the function $\bar{M}(\gamma)$ with respect to the error rectangle $R_j$. If $E(\gamma_j) \geqslant 0$ it means that the curve of the function $\bar{M}(\gamma)$ intersects the error rectangle $R_j$, whose shifting upwards or downwards by $E(\gamma_j)$ does not affect this property (Fig. 3).

(b) Percentage index of the number of measurement points the error rectangles of which are disconnected from the diagram of the function $\bar{M}(\gamma)$

$$H = \frac{100}{2n}\sum_1^n (1 - \operatorname{sgn} E(\gamma_j))$$

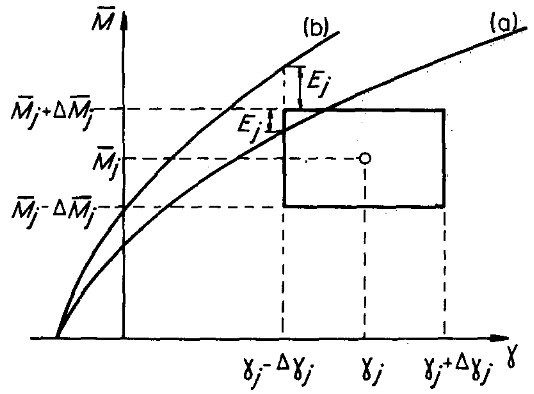


*Figure 3* A scheme of the denotations of an error rectangle relating to the strain hardening curves (a) and (b).

If $H = 0$ it means that the curve of the function $\bar{M}(\gamma)$ intersects the error rectangles of all the measurement points.

(c) The mean approximation error is

$$A = \frac{100}{n}\sum_1^n \frac{|M_j^{\mathrm{T}} - \bar{M}_j|}{\bar{M}_j}$$

and the mean measurement error is

$$B = \frac{100}{n}\sum_1^n \frac{\Delta\bar{M}_j}{\bar{M}_j}$$

By comparing the values of $A$ and $B$ we shall obtain the complete information about the approximation quantity.

(d) Various statistics tests, e.g. $\chi^2$ test for the quantity $SS$ or the series test for $M_j^{\mathrm{T}} - \bar{M}_j$ where by $SS$ we have denoted the following sum

$$SS = \sum_1^n \omega_j\,[M_j^{\mathrm{T}} - \bar{M}_j]^2$$

To solve the problems presented above and to enumerate the discussed approximation quantity measures a program for a digital computer has been prepared. To determine the function minimum the MINUITS program has been applied. E.g. for $n = 57, p = 12, q = 35$ all the calculations took up about 20 sec of the central processor (CP) operation of the digital computer CYBER 70.

## 4. Experimental investigations

The above presented method has been verified by way of example on the strain hardening of nickel of 99.95% purity. This metal was used to produce test-pieces $\phi$ 10 mm x 20 mm, which before the torsion test, were subjected to heat treatment at

TABLE I

| Shear strain on the ranges boundary | | Measures | | | | |
|---|---|---|---|---|---|---|
| | | $A$ | $H$ | $S$ | $SS$ | $B$ |
| with $\gamma_{II/III} = 2.4875$ | $\gamma_{I/II}$ | | | | | |
| | 0.3111 | 0.3427 | 8.7719 | 0.9561 | 42.966 | |
| | 0.3885 | 0.3340 | 3.5088 | 0.8990 | 37.985 | |
| | 0.5042 | 0.3862 | 3.5088 | 1.0221 | 49.103 | |
| | | | | | | 0.8113 |
| with $\gamma_{I/II} = 0.3885$ | $\gamma_{II/III}$ | | | | | |
| | 2.2687 | 0.3306 | 3.5088 | 0.8902 | 37.246 | |
| | 2.4148 | 0.3253 | 3.5088 | 0.8822 | 36.575 | |
| | 2.5603 | 0.3522 | 8.7719 | 0.9453 | 42.000 | |

recrystallization temperature to obtain a homogeneous grain size of 45 $\mu$m. During the torsion test 57 measurement points were registered within the range $10° \leqslant \alpha \leqslant 720°$, the experimental conditions securing the following accuracies:

$$\Delta d = 0.01\,[\mathrm{mm}] \qquad \Delta\alpha_j = 2\,[°]$$

$$\Delta l = 0.1\,[\mathrm{mm}] \qquad \Delta M_j = 0.5\,[\%]$$

The above data enabled determination of the magnitude of the rectangles of the measurement errors. The test was carried out on a testing machine at an angle of twist rate $\dot{\alpha} = 19\,[°/\mathrm{min}]$.

At the beginning the approximation of the strain hardening curve with two ranges was considered. The approximation quality measures were as follows:

$$H = 47.3684$$

$$A = 0.7998$$

$$SS = 232.245$$

Thus we found it necessary to introduce the separation into three ranges. In that case a very good approximation was obtained:

$$H = 1.7544 \qquad \gamma_{I/II} = 0.5042$$

$$A = 0.2156 \quad \text{with} \quad \gamma_{II/III} = 1.9747$$

$$SS = 18.932$$

$$c_{I} = 2.8523$$

$$\text{and} \quad c_{II} = 1.2919$$

$$c_{III} = 0.0597$$

The values of the parameter $c$ calculated in that way are not justified from the physical point of view and the differences in the values are responsible for the fact that with a continuous increase of $\gamma$ the deformation measure $z$ is discontinuous. Thus with further calculations we decided to determine only one value of the parameter $c$ for the entire strain hardening curve under consideration.

Repeated calculations with this assumption helped to state that the best approximation is obtained for $c = 0.74$ and then:

$$H = 3.5088$$

$$A = 0.3173$$

$$SS = 33.108$$

It has also been found that the values of $\gamma_{I/II} = 0.3885$, $\gamma_{II/III} = 2.4148$ remain constant for $0.4 \leqslant c \leqslant 0.8$. Table I presents the procedure (for $c = 0.58$) which has been used when determining the boundaries $\gamma_{I/II}, \gamma_{II/III}$.

Let us note that the derived value $c = 0.74$ may appear doubtful from the point of view of the theory of plasticity. With this in mind the experimental data from the torsion test have been recalculated taking into consideration, besides the approximation quality criterion, also a criterion of analogous process of a unit hardening work in the compression and torsion tests [2]. The corresponding results are given in Table II.

In that case a worse but still "correct approximation" of the torsion test results has been obtained, where:

$$H = 7.0175$$

$$A = 0.3385 \qquad \text{and} \qquad B = 0.8113$$

$$SS = 35.901$$

the material parameters from the torsion and compression tests showing satisfactory agreement. In Fig. 4 theoretical lines have been drawn which correspond to the equations containing the material parameters given in Table II.

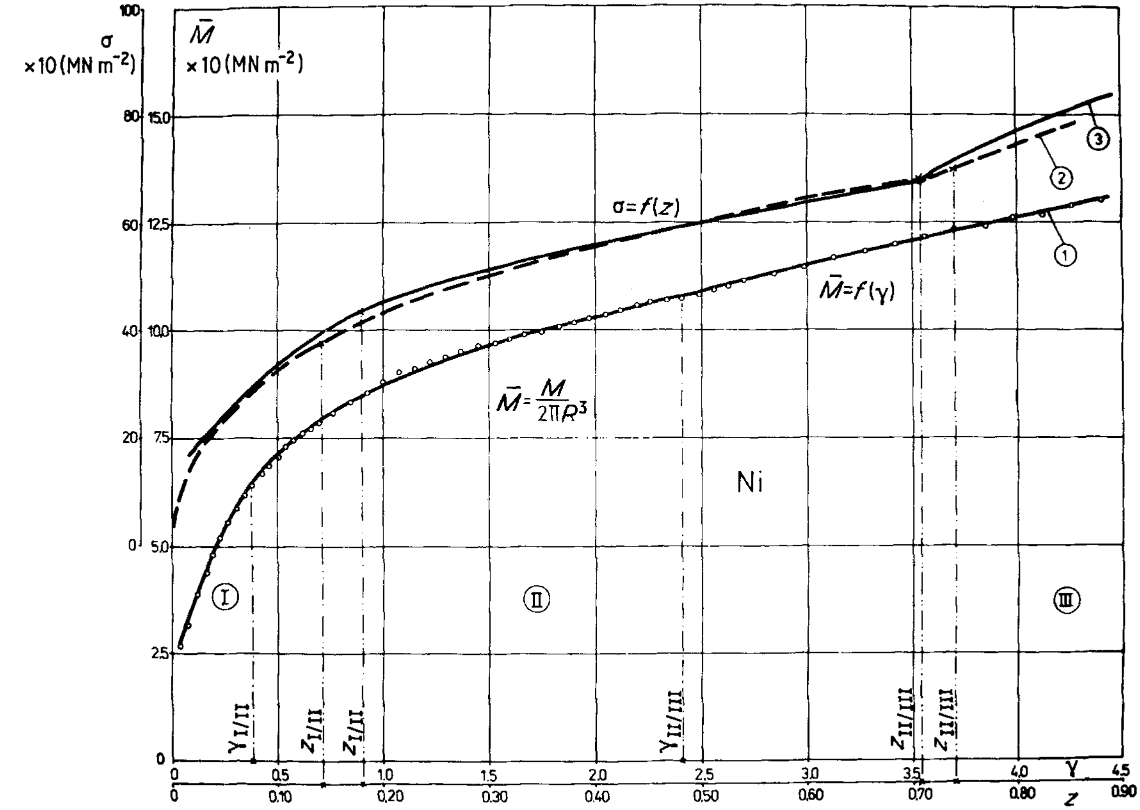


*Figure 4* Strain hardening curves for nickel: 1 – from a torsion test in the system $\gamma, \bar{M}$ against the measurement points, 2 – from a compression test in the system $z, \sigma$, 3 – from a torsion test recalculated onto the systems $z, \sigma$ for $c = 0.51$.

## 5. Conclusions

1. The proposed method enables to determine in an unambiguous way all the parameters of Krupkowski's function (Equation 2). It is especially useful for an analysis of the occurrence of the strain hardening ranges and the determination of the corresponding boundaries.

2. When investigating the strain hardening of nickel it has been found that the parameter $c$ assumes physically justified values if, when determining its value, additional data $\sigma = f(z)$ obtained from another test are taken into consideration.

3. Recalculations of the above data have shown that if the parameter $c$ is determined from the criterion of an analogous strain hardening process in the torsion and compression tests, the values of the other parameters in both these tests are similar.

4. The relationship $\tau/\sigma$ determined in this way for the investigated metal was 0.51, thus it corresponds to the Tresci criterion.

TABLE II

| Material parameter | Torsion test | | | Compression test | | |
|---|---|---|---|---|---|---|
| | Strain hardening range | | | Strain hardening range | | |
| | I | II | III | I | II | III |
| $k \times 10$ MN m$^{-2}$ | 101.57 | 75.36 | 92.49 | 92.62 | 77.41 | 90.75 |
| m | 0.5076 | 0.3336 | 0.8657 | 0.4623 | 0.3670 | 0.9037 |
| $\gamma^1$ | | 0.0207 | | | – | |
| $z^1$ | | (0.0105) | | | –0.0005 | |
| $c$ | | 0.51 | | | – | |
| $\gamma_{I/II}$; $\gamma_{II/III}$ | 0.3885 | 2.4148 | | – | – | |
| $z_{I/II}$; $z_{II/III}$ | (0.1797) | (0.7082) | | 0.1430 | 0.7401 | |

## References


1. CH. DUGUET, "Déformation des Corps Solides. Limite d'Élasticité et Resistance à la Rupture" Première Partie. Statique Speciale. (Paris, 1882) p. 213.
2. A. KRUPKOWSKI, *Bull. Acad. Pol. Sci. Sér. Sci. Tech.* **XVII** (1969) 67.
3. *Idem, Annls. Acad. Sci. Tech. Varsovie* 7 (1939–45) 113.
4. A. KRUPKOWSKI and W. TRUSZKOWSKI, *Bull. Acad. Pol. Sci. Sér. Sci. Tech.* **XV** (1967) 1.
5. A. KRUPKOWSKI, A. DAŃDA, A. GRABIANOWSKI and S. WIERZBIŃSKI, *ibid.* **XXVI** (1978) 25.
6. W. TRUSZKOWSKI, *Archwm. Hutn.* **26** (1981) 395.
7. A. KRUPKOWSKI and A. GRABIANOWSKI, *Bull. Acad. Pol. Sci. Sér. Sci. Tech.* **XVIII** (1970) 45.
8. A. GRABIANOWSKI, *Archwm. Hutn.* **27** (1982) 225.
9. A. KRUPKOWSKI, A. GRABIANOWSKI and A. DAŃDA, *ibid.* **21** (1976) 515.



*Received 10 August*
*and accepted 27 September 1982*